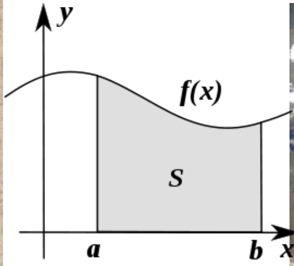

The art of searching for extremes from Euclid to Dantzig: A historical pursuit of optimisation theory as a basis for the evolution of optimisation methods of water resources management

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ΥΔΡΟΚΡΙΤΗΣ HYDROCRITES
ΠΑΝΕΠΙΣΤΗΜΙΟ ΠΑΤΡΩΝ UNIVERSITY OF PATRAS
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TUM

OVERVIEW

- **Introduction -** in optimisation concept
  - in history of water allocation

- **Historical spoors of optimisation theory:**
  Representative optimisation problems

1. **Geometric optimisation:**

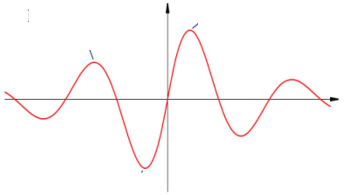
   Milestones of optimisation thinking in ancient world

2. **Birth and evolution of mathematical optimisation theory**:
   Calculus of variation

3. **20th century:**

   Optimisation – War, Linear programming

INTRODUCTION

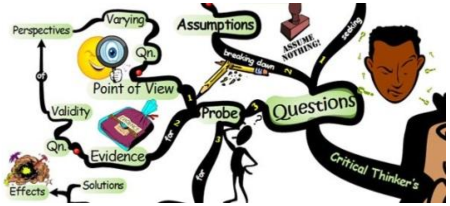


- **<u>Exploration of minima and maxima</u>:**

  ❖ **Optimizing:** → ***(A)*** Simple everyday activities → ***(B)*** Complicated problems

  ❖ **Desire of optimality** ⟶ Inherent for humans

  ❖ **Rationality: "Best decision"** based on possible alternatives



- **<u>In mathematics</u>:**

Most honey using smallest comb

Greatest profit for least expense

Greatest volume for least area

Greatest area for the least perimeter

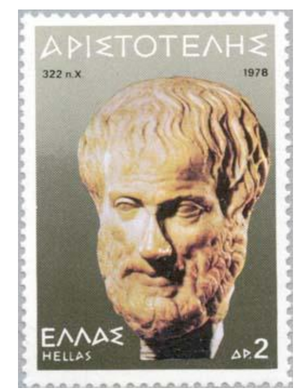
INTRODUCTION

- **<u>Optimization in nature</u>:**

  - ❖ **Principle of least action:**

    “If there occurs some changes in nature, the amount of action necessary for this must be as small as possible.”

    - ***P.L.M.de Maupertius*** (Essai de cosmologie, 1750)
    - ***L.Euler*** (*M*ethods of finding curves that are subject to a maximum or minimum property, 1743)



- **<u>Stepping beyond the strict mathematical view</u>:**
  - ❖ Art of personal sufficiency
  - ❖ Enjoying the optimum amount of anything
  - ❖ **Golden Mean:** “Path to contentment lies between *twin evils* of having *too much* or *too little*” in Nicomachean Ethics

HISTORICAL SPOORS OF OPTIMISATION THEORY

WATER RESOURCES MANAGEMENT – WATER ALLOCATION

Egypt

New Kingdom
Middle Kingdom
Old Kingdom
Early Dynasty
Naqada
Pre Dynasty

Mesopotamia

Old Babylonian Empire
Akkadian Empire
Early Dynasty 1
Jamdel Nasr
Uruk
Ubaid
Preceramic

Indus Valley

Mauryan Empire
Late Harappan
Harappan
Early Harappan
Balakot
Mehrgarh IIB
Mehrgarh IIA

China

China Empire
Chou
Shang
Hsla
Longshan
Yangshoa
Neolithic

HISTORICAL SPOORS OF OPTIMISATION THEORY

OPTIMAL WATER ALLOCATION

- ❑ **19th Century: Optimisation of constrained resources allocation**

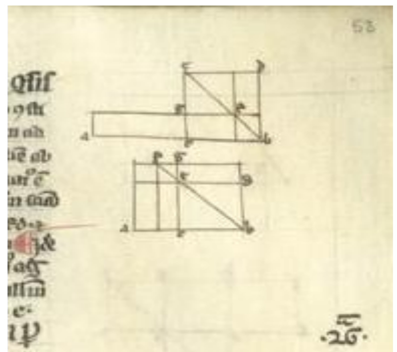
- ❑ **Optimal water allocation scheme to each water user under time-varying inflows, demands and cost-benefit functions**

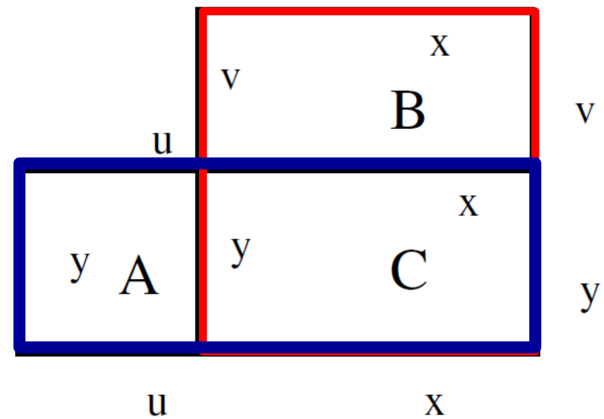
HISTORICAL SPOORS OF OPTIMISATION THEORY

1. GEOMETRIC OPTIMISATION: Milestones in ancient world

- **Euclid, 300BC: Isoperimetric theorem for square**

Square: greatest area among rectangles of equal parameter





✓ **Same perimeter:** $2x + 2u + 2y = 2x + 2y + 2v$

$\longrightarrow u = v.$

✓ **Rectangular: equal sides**: $x = y + v$

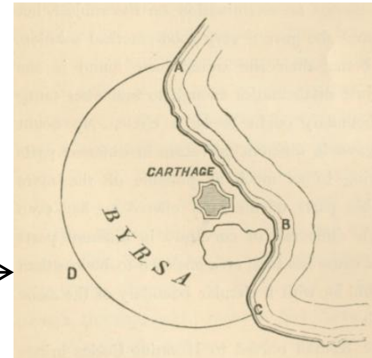
✓ **Area of B:** $vx = v(y+v) = u(y+v) = uy + ux$

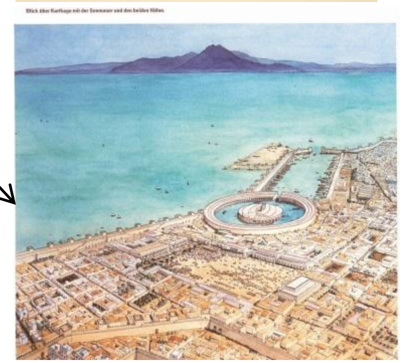
✓ **Area of rectagular** $<$ **Area of square**

HISTORICAL SPOORS OF OPTIMISATION THEORY

1. GEOMETRIC OPTIMISATION: Milestones in ancient world

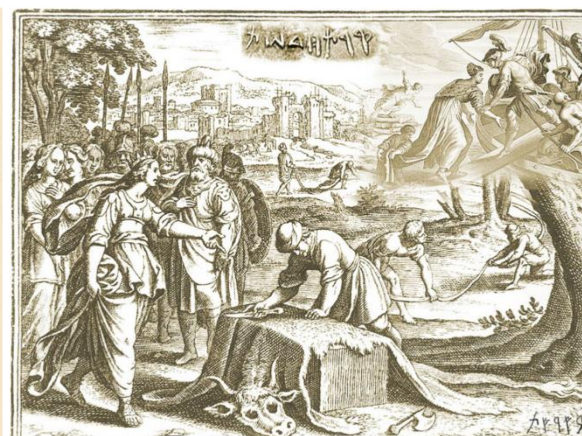
- **Queen Dido's Problem**





Byrsa Tombs at Carthage and view of Goletta, Tunisia, circa 1899

Rembrandt's "Dido Divides the Oxhide" (mid-1600s)



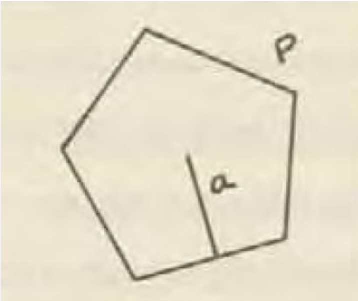
HISTORICAL SPOORS OF OPTIMISATION THEORY

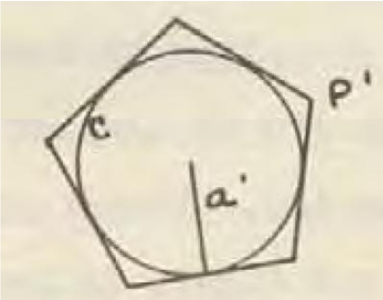
1. GEOMETRIC OPTIMISATION: Milestones in ancient world

- **Zenodorus, 200 BC: Queen Dido's Problem**

  Shape for the greatest possible area with perimeter of given length

- **C:** circle of perimeter **p**
- **P:** regular polygon of equal perimeter
- **P‘:** polygon circumscribing **C** similar to **P**
- **a and a‘:** apothems of **P** and **P‘** and **a‘** radius of the circle





- From polygon similarity: a/a‘ = p/p‘
- Since p‘ > p $\longrightarrow$ a‘ > a
- Based on Archimedes‘ theorem: = ½ of area of a rectangle with length = perimeter and width=radius of C

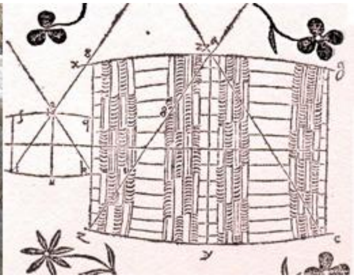
Area of C = a‘p/2

Area of P =ap/2

HISTORICAL SPOORS OF OPTIMISATION THEORY

2. BIRTH & EVOLUTION OF MATHEMATICAL THEORY OF OPTIMISATION

- **<u>Important problem related to calculus</u>:**



  - **Keppler, 1615:**
  - **Best proportion of wine barrel for max volume:**

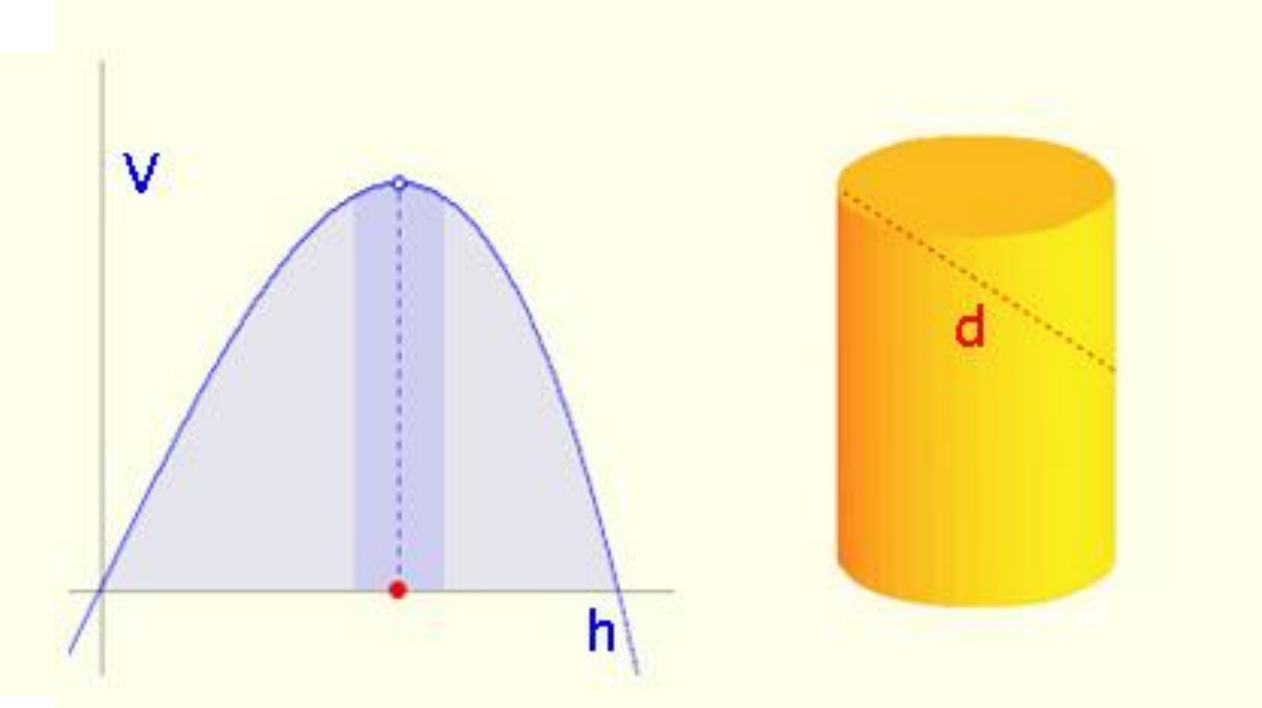
$$3h^2 = 4d^2$$

STEREOMETRIA DO-

De virga cubicâ eiusq; certitudine.

THEOREMA XXVI.

In dolijs, quæ sunt inter se figuræ similis; proportio capacitatum est tripla ad proportionem illarum longitudinum, quæ sunt ab orificio summo, ad imum calcem alterutrius Orbis lignei.



HISTORICAL SPOORS OF OPTIMISATION THEORY

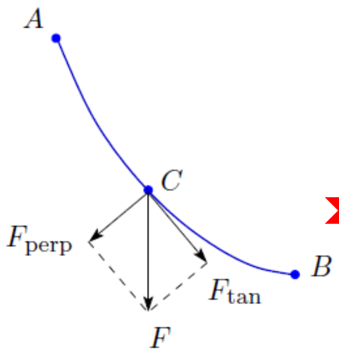
2. BIRTH & EVOLUTION OF MATHEMATICAL THEORY OF OPTIMISATION

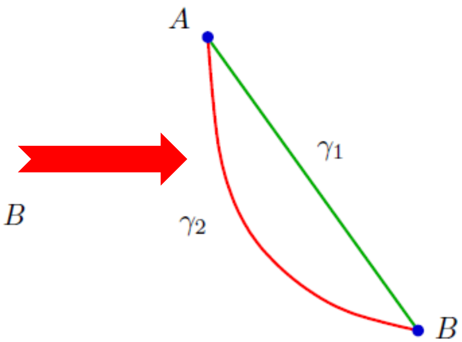
- **17th** century: **Calculus of variations**

- **<u>Brachistochrone problem or the curve of fastest descent</u>:**

  - Find **curve shape** for which a bead **slips** from rest and by gravity from one point to another in the **least time**.

  - Greek (brachistos) "the shortest" and (chronos) "time





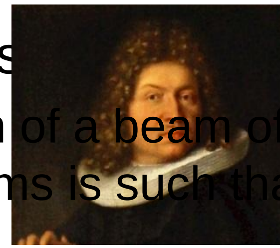
HISTORICAL SPOORS OF OPTIMISATION THEORY

2. BIRTH & EVOLUTION OF MATHEMATICAL THEORY OF OPTIMISATION

Brachistochrone problem or the curve of fastest descent:

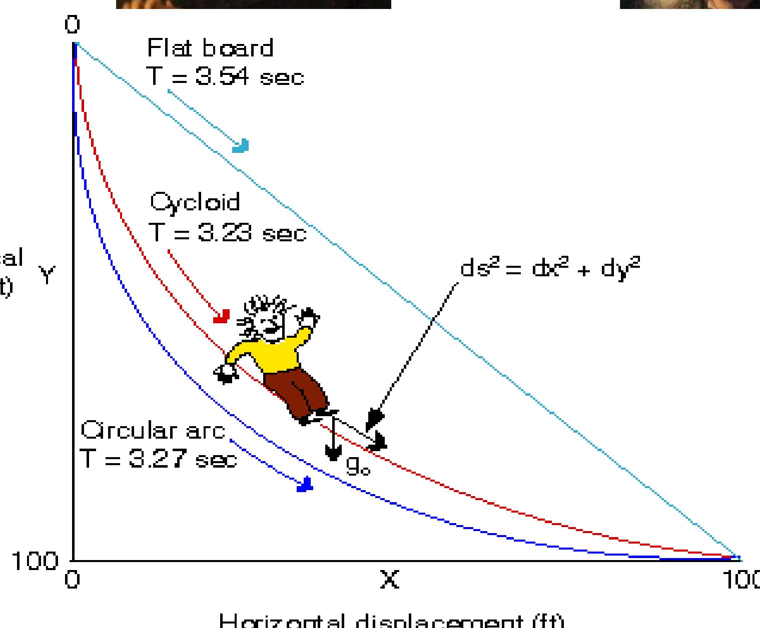
- Johann Bernoulli's solution: Snell's Law

"Change in direction of a beam of light as it crosses the boundary between two mediums is such that sinθ1 / sinθ2 is constant."



Newton

Johann B

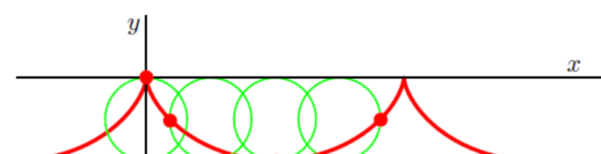


Gottfried

Guillaume de l'Hôpital

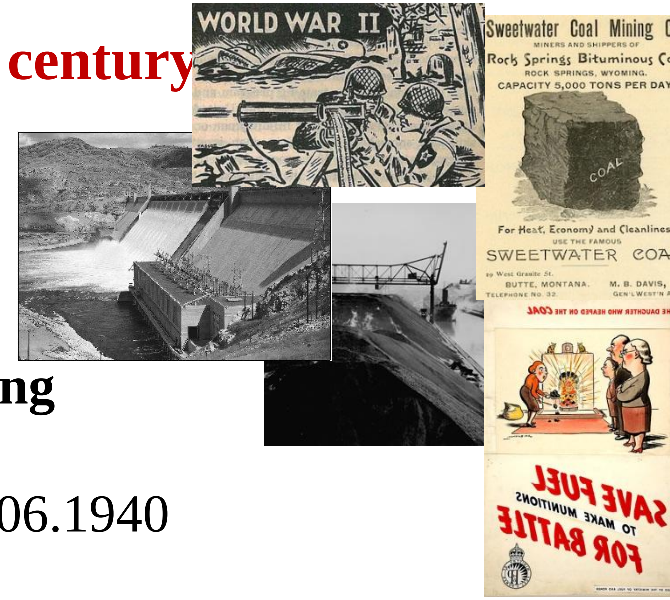
Cycloid:

curve traced by a point on the circumference of a circle as the circle rolls along a straight line.



HISTORICAL SPOORS OF OPTIMISATION THEORY

3. Evolution of optimisation methods in the 20th century



- **<u>Optimization - war</u>:**

  - **<u>Pierre Massé, 1944</u>:**„le jeu des réservoir“

    - ✓ **Dynamic, stochastic and recursive programming**
    - ✓ Second World War: France-Germany: 09.1939 – 06.1940
    - ✓ Monthly water abstractions from reservoir for minimizing current and future coal use
    - ✓ Dammed reservoir with uncertain precipitation → explore rules for regulating optimal reservoirs flows
    - ✓ **Conditional policy**: policy for each stage based on both uncertain new inflows and decisions of previous stage
    - ✓ Image of fork  **Decision tree**

HISTORICAL SPOORS OF OPTIMISATION THEORY

3. Evolution of optimisation methods in the 20th century

- **<u>Optimization and war</u>:**

  - **<u>Linear optimisation</u>:**

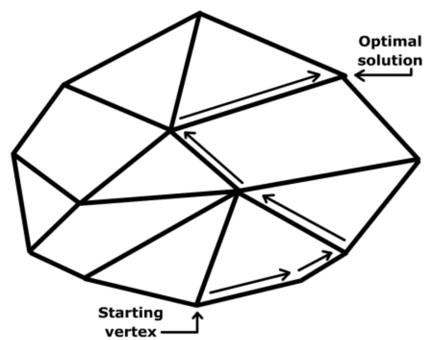
$$\begin{array}{ll} \text{maximize} & \mathbf{c}^{\mathrm{T}}\mathbf{x} \\ \text{subject to} & A\mathbf{x} \leq \mathbf{b} \\ \text{and} & \mathbf{x} \geq \mathbf{0} \end{array}$$

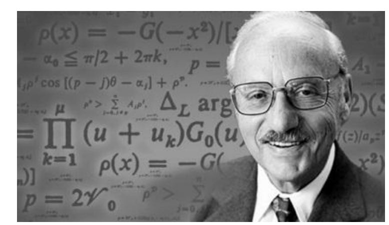
Leonid Kantorovich, 1939

- ✓ earliest linear programming problems
- ✓ for use during World War II
- ✓ to plan expenditures and returns
- ✓ in order to reduce costs to the army and increase losses to the enemy.

George Dantzig, 1947

- ✓ Simplex method





John von Neumann, 1947

- ✓ theory of duality as a linear optimization solution

THANK YOU FOR YOUR ATTENTION!